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Invariants on primary abelian groups and a problem of Nunke

Abstract: If G is an arbitrary abelian p-group, an invariant K_G is defined which measures how closely G resembles a direct sum of cyclic groups. This invariant consists of a class of finite sets of regular cardinals, and is inductively constructed using filtrations of various subgroups of G. Moreover, K_G can also be considered to be a measure of the presence of non-zero elements of infinite height in G. This construction is particularly useful when the group has final rank less than the smallest weakly Mahlo cardinal; and in this case, G is a direct sum of cyclics if and only if K_G is empty. These deliberations are then used to place several of the most significant results relating to direct sums of cyclics into a significantly broader context. For example, G is shown to be almost a direct sum of cyclics if and only if every set in K_G has at least two elements. Finally, K_G is used to give a more complete and concrete answer to a classical problem of Nunke, which asks when the torsion product of two abelian p-groups is a direct sum of cyclics.