SEMINARI DE PROBABILITATS

PROBABILITY SEMINAR

Universitat de Barcelona–Universitat Autònoma de Barcelona

Titles and abstracts for the academic year 2008-2009

01/10/2008 Eddy Mayer-Wolf, Technion, Israel.
Correlation Inequalities in Wiener Space.

Abstract We present a number of correlation inequalities in finite or infinite dimensional Gaussian spaces involving suitably monotone, convex or log-concave functionals, which are presented in a unified manner as direct consequences of a suitable covariance expansion in terms of the Ornstein-Uhlenbeck semigroup. This is joint work with A.S. Ustunel and M. Zakai.

08/10/2008 Sebastian del Baño, CRM, Bellaterra, Spain.
Spot inversions in the Heston models.

Abstract We analyse the Heston stochastic volatility model under an inversion of spot. The result is that under the appropriate measure changes the resulting process is again a Heston type process whose parameters can be explicitly determined from those of the original process. This behaviour can be interpreted as some measure of 'sanity' of Heston model but does not seem to be a general feature of stochastic volatility processes.

29/10/2008 Josep Vives, Universitat de Barcelona, Spain.
An anticipating Itô formula for Lévy processes.

Abstract In this paper, we use the Malliavin calculus techniques to obtain an anticipative version of the change of variable formula for Lévy processes. Here the coefficients are in the domain of the anihilation (gradient) operator in the 'future sense', which includes the family of all adapted and square-integrable processes. This domain was introduced on the Wiener space by Alòs and Nualart (1998). Therefore, our Itô formula is not only an extension of the usual adapted formula for Lévy processes, but also an extension of the anticipative version on Wiener space obtained in Alòs and Nualart (1998).

05/11/2008 Marta Sanz-Solé, Universitat de Barcelona, Spain.
Hitting probabilities for stochastic waves.

Abstract For $\mathbb{R}^d$-valued stochastic processes $\{v(x), x \in \mathbb{R}^m\}$, we shall discuss conditions providing lower and upper bounds for the hitting probabilities $P\{v(I) \cap A \neq \emptyset\}$ in terms of the capacity and the Hausdorff measure of A, respectively. Applications to the stochastic wave equation with additive correlated noise will be given. The results are part of ongoing work with R. Dalang.

12/11/2008 Sebastian del Baño, CRM, Bellaterra, Spain.
Some considerations in the trading of exotic options.

Abstract We present some of the real life features of the FX spot and derivatives market and show some consequences these have in the mathematical modelling of the market.

19/12/2008 Arturo Kohatsu-Higa, Osaka University, Japan.
An Operator Approach for Markov Chain Weak Approximations with an Application to Infinite Activity Levy Driven SDEs.

Abstract Weak approximations have been developed to calculate the expectation value of functionals of stochastic differential equations, and various numerical discretization schemes (Euler, Milshtein) have been studied by many authors. We present a general framework based on semigroup expansions for the construction of higher order discretization schemes and analyze its rate of convergence. We also apply it to approximate general Lévy driven stochastic differential equations.

19/12/2008 Giovanni Peccati, Université Paris Ouest, Nanterre La Défense, France.
Weak convergence on the Poisson space: decoupling, Stein's method and low influences.

Abstract We will describe how to obtain limit theorems for non-linear functionals of a Poisson measure, by combining Malliavin calculus, decoupling and Stein's method. We will evoke some applications to prior specification in Bayesian nonparametric survival analysis. Also, we will enlight some links with the notion of "kernel with low influences", which is a concept related to invariance principles for polynomial functionals of i.i.d. sequences. The core of the talk is based on a joint work with J.-L. Solé, F. Utzet and M.S. Taqqu.

07/01/2009 Eulàlia Nualart, Université Paris 13, Villetaneuse, France.
Minoració de la densitat d'una variable aleatòria no degenerada en l'espai de Wiener.

Abstract Obtenim una minoració de la densitat d'una variable aleatòria d-dimensional no degenerada en l'espai de Wiener sota una condició del moment exponencial de la divergència de çovering vector fields". Aquest treball ha estat realitzat conjuntament amb el Professor Paul Malliavin.

14/01/2009 Mario Wschebor, Universidad de la República, Uruguay.
The distribution of the maximum of a real-valued random field. Second order propierties.

Abstract Let $\chi := \{X(t); t \in S\}$ be a real-valued random field on some parametre set S, and $M := \sup_{t\in S} X(t)$ its supremum.

For Gaussian processes, a number of basic inequalities for $P(M > u)$ have been known since the 1970's (Dudley, Landau-Shepp, Fernique, Borell, Sudakov-Tsirelson). These are essential for the development of most of the mathematical theory. However, in a wide number of applications, the general situation is that these inequalities are not good enough, since they

depend on certain constants that one is unable to estimate or for which estimations differ substantially from the true values and also, they are very inaccurate for certain classes of processes.

Since the 1990's several methods have been intriduced with the aim of obtaining more precise results than those arising from the classical theory, at least under certain restrictions on the process χ. These results are interesting both from the standpoint of the mathematical theory and of their use in significant applications. The restrictions on χ include the requirement that the domain S have some finite-dimensional geometrical structure and the paths of the random field, a certain regularity.

More precisely, one wants to write, whenever it is possible,

$$P(M > u) = A(u) \exp\left(-\frac{1}{2}\frac{u^2}{\sigma^2}\right) + B(u) \tag{1}$$

where $A(u)$ is a known function having polynomially bounded growth as $u \to +\infty$, $\sigma^2 = \sup_{t\in S} Var(X(t))$ and $B(u)$ is an error bounded by a centered Gaussian density with variance $\sigma_1^2, \sigma_1^2 < \sigma^2$. We will call the first (respectively the second) term in the right-hand side of (1) the 'first (resp second) order approximation of $P(M > u)$.'

The aim of the talk is to present recent results with more precise approximations of the tails of the distribution of the random variable M. We will be especially interested in the approximation of $P(M > u)$ for large u, but we also give results that can be used for all u. The content of the talk is a part of Chapters 7 and 8 of the book 'Level sets and extrema of random processes and fields' by Jean-Marc Azaïs and Mario Wschebor, John Wiley and Sons, to appear in March 2009.

21/01/2009 Khalifa Sebaiy. Université Cadi Ayyad de Marrakech i Departament de Matemàtiques, UAB.

Occupation densities for certain processes related to fractional Brownian motion.

Abstract We establish the existence of a square integrable occupation density for two classes of stochastic processes. First we consider a Gaussian process with an absolutely continuous random drift, and secondly we handle the case of a (Skorohod) integral with respect to the fractional Brownian motion with Hurst parameter $H > 1/2$. The proof of these results uses a general criterion for the existence of a square integrable local time, which is based on the techniques of Malliavin calculus. This is joint work with Y. Ouknine, D. Nualart and C. Tudor.

04/02/2009 Ely Merzbach, Bar-Ilan University, Israel.

The Set Indexed Lévy Process.

Abstract We present a satisfactory definition of the important class of Lévy Processes indexed by a general collection of sets (SILP: Set-Indexed Lévy Process). This class is characterized by the Lévy-Khintchine representation. The notion of increment stationarity for

set-indexed processes is discussed. Another characterization is presented using projections on flows. Connection between SILP and the set-indexed Markov property will be given. Finally the Set-Indexed Compound process will be studied as a typical example of SILP.

18/02/2009 Rosario Delgado, UAB.

Asymptotics for the maximum of a linear combination of the components of a rfBm process, with application to queueing networks.

Abstract Let $W = \{W(t),\, t \geq 0\}$ be a $J-$dimensional rfBm process on the first orthant S with drift vector $\theta \in S$ and Hurst parameter $H \in (\frac{1}{2},\, 1)$. Let $a_1,\, \ldots,\, a_J$ be positive real numbers, and define

$$M(t) = \max_{0 \leq s \leq t} \sum_{j=1}^{J} a_j\, W_j(s) \tag{2}$$

We prove that the increase of $M(t)$ as $t \to \infty$ is closer and not bigger than that of a function $f(t)$, which is

$$f(t) = \begin{cases} t^H\, (\log t)^{\frac{1}{2-2\,H}} & \text{in the drift-less case } (\theta = 0) \\ t\, (\log t)^{\frac{1}{2-2\,H}} & \text{otherwise.} \end{cases}$$

The study of (2) is motivated by an application to queueing networks: consider a multi-class queueing network with K fluid classes, and J stations which have a single server and an infinite buffer at each one, with $K \geq J \geq 1$, and such that each server can process one or more fluid classes (but each fluid class can be processed at only one station). Feedback is allowed and a FIFO and non-idling service discipline is assumed. If the arrival process is generated by a large enough number of heavy-tailed On/Off sources, under some conditions including mainly *heavy traffic* (or asymptotical criticality) and *state space collapse*, the limits of the ($J-$dim.) workload and the ($K-$dim.) fluid in queue processes, conveniently normalized, W and Z respectively, exist, and they are related by means of a "lifting"matrix Δ. Moreover, W turns out to be a drift-less rfBm process on S with Hurst parameter in $(\frac{1}{2},\, 1)$. Therefore, $\max_{0 \leq s \leq t} \sum_{k=1}^{K} Z_k(s)$ can be interpreted as the total amount of fluid in queue in the system on the interval $[0,\, t]$, and coincides with $M(t)$ for some specific weights $a_1,\, \ldots,\, a_J$ given by Δ.

25/02/2009 Ciprian Tudor, Université Paris 1, France.

Limits of the wavelet variations of self-similar processes.

Abstract The purpose is to make a wavelet analysis of self-similar stochastic processes by using the techniques of the Malliavin calculus and the chaos expansion into multiple stochastic integrals. Our examples are the fractional Brownian motion and the Rosenblatt process. We study the asymptotic behavior of the statistics based on the wavelet coefficients of these processes.

25/02/2009 Jorge León, CINVESTAV, Mexico.

Ecuaciones con retraso en el sentido de Young.

Abstract En esta charla estudiaremos la existencia y unicidad de soluciones de ecuaciones diferenciales con retraso gobernadas por una funcion Hölder continua con parametro mayor que 1/2. La integral involucrada es en el sentido de Young.

18/03/2009 Joan-Andreu Lázaro-Camí, Centre de Recerca Matemàtica.
Antithetic variates in higher dimensions.

Abstract We introduce the concept of multidimensional antithetic as the absolute minimum of the covariance function $O(N) - - > R$ defined by $A - - > Cov(f(\xi),\ f(A\xi))$ where ξ is a standard N-dimensional normal random variable and $f : R^N - - > R$ is an almost everywhere differentiable function. The antithetic matrix is designed to optimise the calculation of $E[f(\xi)]$ in a Monte Carlo simulation. We present an iterative annealing algorithm that dynamically incorporates the estimation of the antithetic matrix within the Monte Carlo calculation.

This is a joint work with Sebastian del Baño Rollin.

25/03/2009 Bohdan Maslowski. Academy of Sciences, Check Republic, Prague.
Infinite Time Horizon Stochastic Control for SPDEs.

Abstract Controlled Stochastic PDE's with the cost functional of discounted and ergodic types will be considered and some recent results obtained jointly with Ben Goldys on the ergodic control will be outlined. The optimal cost and optimal control in these cases may be found in a feedback form by means of the solutions to appropriate stationary HJB equations in a Banach spaces, the existence and uniqueness of which has been proved. The results are applicable to controlled stochastic reaction-diffusion equations.

01/04/2009 Lluís Quer i Sardanyons, CRM, UAB.
Gaussian density estimates for solutions to quasi-linear SPDEs.

Abstract We establish lower and upper Gaussian bounds for the solutions to the heat and wave equations driven by an additive Gaussian noise, using the techniques of Malliavin calculus and recent density estimates obtained by Nourdin and Viens in *Density estimates and concentration inequalities with Malliavin calculus, preprint.* In particular, we deal with the one-dimensional stochastic heat equation in [0,1] driven by the space-time white noise, and the stochastic heat and wave equations in $\mathbb{R}^d$ ($d \geq 1$ and $d \leq 3$, respectively) driven by a Gaussian noise which is white in time and has a general spatially homogeneous correlation.

These results have been obtained in collaboration with D. Nualart (University of Kansas).

22/04/2009 Albert Ferreiro-Castilla, CRM, UAB.
A new look at the Heston characteristic function.

Abstract A new expression for the characteristic function of log-spot in Heston model is presented. This expression more clearly exhibits its properties as an analytic characteristic function and allows us to compute the exact domain of the moment generating function. We

also give a factorization of the moment generating function as product of Bessel type factors, and an approximating sequence to the law of log-spot is deduced. This is joint work with Sebastian del Baño Rollin and Frederic Utzet.