

BREAKING THE DUALITY IN THE RETURN TIMES THEOREM

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Bourgain proved the following Return Times theorem: Let $\mathbf{X} = (X, \Sigma, \mu, \tau)$ be a dynamical system. Let also $1 \leq p, q \leq \infty$ be such that $\frac{1}{p} + \frac{1}{q} \leq 1$. For each function $f \in L^p(X)$ there is a universal set $X_0 \subseteq X$ with $\mu(X_0) = 1$, such that for each second dynamical system $\mathbf{Y} = (Y, F, \nu, \sigma)$, each $g \in L^q(Y)$ and each $x \in X_0$, the averages

$$\frac{1}{N} \sum_{n=0}^{N-1} f(\tau^n x) g(\sigma^n y)$$

converge ν - almost everywhere. We show how to break the duality in this theorem. More precisely, we prove that the result remains true if $p > 1$ and $q \geq 2$. We emphasize the strong connections between this result and the Carleson-Hunt theorem on the convergence of the Fourier series. We also prove similar results for the analog of Bourgain's theorem for series, where no positive results were previously known. This is joint work with Michael Lacey, Terence Tao and Christoph Thiele.