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GAGLIARDO–NIRENBERG - TYPE INEQUALITIES
AND ESTIMATES OF THE MODULI OF CONTINUITY

We study multiplicative inequalities of Gagliardo–Nirenberg - type which connect partial moduli of continuity and partial derivatives of functions with respect to a fixed variable in different Lorentz norms. The main results are expressed in the estimates of the following type

$$\left(\int_{\delta}^{\infty} [h^{-\theta r} \omega_j^r(f; h)_{p,s}]^s \frac{dh}{h} \right)^{1/s} \\ \leq c \|f\|_{p_0, s_0}^{1-\theta} [\delta^{-r} \omega_j^r(f; \delta)_{p_1, s_1}]^{\theta}$$

where $0 < \theta < 1$,

$$\frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}, \quad \frac{1}{s} = \frac{1-\theta}{s_0} + \frac{\theta}{s_1},$$

and indices p_i, s_i satisfy certain conditions. In particular, from these estimates we derive optimal inequalities involving Besov norms and Lorentz norms. We study also the limiting case $p_1 = s_1 = 1$ and estimates in terms of the total variation.