

THE BRAKE-TO-SYZYGY MAP

RICHARD MONTGOMERY

(University of California at Santa Cruz)

In the three-body problem, we study solutions with zero initial velocity (brake orbits). Following such a solution until the three masses become collinear (syzygy) defines a kind of Poincaré map which we call the “syzygy map”. Viewed after reduction, the syzygy map is a map from one punctured open two-dimensional disc to another, the punctures corresponding to the Lagrange triple collision orbit. Among the results we discuss are the following. For most masses, the map extends continuously to the punctures. The image of the map contains the binary collision locus. These binary collision brake orbits can be realized Jacobi-Maupertuis minimizers. There exists a new periodic brake orbit whose second “hit” with the syzygy locus is orthogonal. The syzygy map grew out of a desire to better understand which syzygy sequences can occur within the zero-angular momentum three-body problem. We will describe this motivation, and what the map tells us about stutters within syzygy sequences. We present several open questions, very simple to state, regarding the nature of the orbits realizing the map. This is joint work with Rick Moeckel and Andrea Venturelli.