# Non-existence of principal values of signed Riesz transforms of non-integer dimension


Aleix Ruiz de Villa

Universitat Autònoma de Barcelona



**Abstract**

We prove that, given $s \geq 0$ and a non-zero Borel measure $\mu$ in $\mathbb{R}^m$, if for $\mu$-almost every $x \in \mathbb{R}^m$ the limit

$$\lim_{\varepsilon \to 0} \int_{|x-y|>\varepsilon} \frac{x-y}{|x-y|^{s+1}} d\mu(y)$$

exists and

$$0 < \limsup_{r \to 0} \frac{\mu(B(x,r))}{r^s} < \infty,$$

then $s$ is an integer. In particular, if $E \subset \mathbb{R}^m$ is a set with positive and finite $s$-dimensional Hausdorff measure $H^s$ and for $H^s$-almost every $x \in E$ the limit

$$\lim_{\varepsilon \to 0} \int_{|x-y|>\varepsilon} \frac{x-y}{|x-y|^{s+1}} dH^s_{|E}(y)$$

exists, then $s$ is an integer.