SEMINARI DE PROBABILITATS
PROBABILITY SEMINAR
Universitat de Barcelona–Universitat Autònoma de Barcelona

Facultat de Matemàtiques, Universitat de Barcelona
Gran Via de les Corts Catalanes, 585; E-08007 Barcelona
Aula/Room IMUB-Facultat de Matemàtiques, 2nd floor
4 p.m.

Titles and abstracts for the academic year 2006-2007

04/10/2006 Antoine Lejay, Institut Élie Cartan, Université Henri Poincaré, Vandoevre-lès Nancy, France
Introduction to rough paths

Abstract The theory of rough paths have proved to be useful tool for stochastic analysis, especially in a context different from the one of semi-martingales. This talk is then an attempt to present the last developments of this theory in an unified way, and to endow the importance of the required algebraic structures from basic considerations on ordinary integrals.

04/10/2006 Jeannette H.C. Woerner, University of Göttingen , Germany
On the sample path behaviour of stochastic processes: the Blumenthal-Getoor index and the Hurst parameter

Abstract For stochastic modeling, specially in finance, turbulence and meteorology, an important feature is the behaviour of the sample paths of the underlying stochastic process. For jump processes the activity of the jumps can be measured in terms of the Blumenthal-Getoor index, whereas for fractional Brownian motion the Hurst parameter determines the regularity.
We propose a class of easily computable estimators simultaneously for the Blumenthal-Getoor index and the Hurst exponent, based on discrete observations. Furthermore, we derive consistency and a distributional result for these estimators and provide a graphical tool to distinguish between purely continuous processes and processes with jumps.

18/10/2006 Daniel Conus, EPFL, Lausanne, Switzerland
Random field solution to the non linear stochastic wave equation in high dimensions

Abstract In 1999, R.C. Dalang extends the stochastic integral with respect to martingale measures developped by Walsh (1984) to be able to integrate

non-negative Schwartz distributions. This extension allows him to find a random-field solution to the 3-dimensional non-linear stochastic wave equation in the case of a noise white in time and correlated in space. Under slightly stronger assumptions, we extend these results to integrate a more general class of Schwartz distributions. In particular, this class contains the fundamental solution of the wave equation in dimensions greater than 3. This leads to a square-integrable random-field solution to the non-linear stochastic wave equation in any dimension with the same noise as above. In the particular case of an affine multiplicative noise, we obtain estimates on p-th moments of the solution ($p > 1$), and we show that the solution is Hlder continuous of the same exponent as in the 3-dimensional case.

08/11/2006 Josep Lluís Solé, UAB, Barcelona, Spain
Sobre les martingales polinomials en espai /temps associades a un procés de Lévy

Abstract En aquest seminari estudiarem els polinomis $P_n(x,t)$ tals que $\{P_n(X_t,t), t \geq 0\}$ és una martingala, on $\{X_t, t \geq 0\}$ és un procés de Lévy. Veurem que apareixen bàsicament els anomenats polinomis dels cumulants. Els relacionarem amb els polinomis de Kailath-Segall i en treurem conseqüències.

22/11/2006 Florin Awram, Université Pau, Pau, France
Some degenerate exit problems of two-dimensional Lévy processes from the quadrant (joint work with Zbigniew Palmowski and Martijn Pistorius

Abstract The one dimensional exact and asymptotic exit/ruin theory have been developed by Cramér and Lundberg as an application of Laplace's method. In several dimensions, this method breaks down; exact solutions are very rarely available, and even asymptotic expansions (beyond the large deviations logarithmic approximation) are not fully studied. We show that however exact analytic and asymptotic results are available for a certain class of "degenerate" multidimensional Lévy processes which appear in proportional reinsurance problems. Their study reduces to one-dimensional first-passage problems with piecewise-linear barriers.

29/11/2006 Florin Awram, Université Pau, Pau, France
On a Szegö type limit theorem for kernel-graph integrals, the Hölder-Young-Brascamp-Lieb inequality, and the asymptotic theory of integrals and quadratic forms of stationary fields

20/12/2006 Marta Sanz-Solé, Universitat de Barcelona, Barcelona, Spain
Analyzing a parabolic SPDE from an analytic and a stochastic point of view

Abstract: We study strictly parabolic stochastic partial differential equations on $\mathbb{R}^d$, $d \geq 1$, driven by a Gaussian noise white in time and coloured in space. Assuming that the coefficients of the differential operator are random, we give sufficient conditions on the correlation of the noise ensuring Hölder continuity for the trajectories of the solution of the equation. For self-adjoint operators with deterministic coefficients, the mild and weak formulation of the equation are related, deriving path properties of the solution to a parabolic Cauchy problem in evolution form. This is a joint work with Marco Ferrante, Padova, Italy.

10/01/2007 Rosario Delgado, Universitat Autònoma de Barcelona, Bellaterra, Spain
El moviment Brownià fraccional amb reflexió com a límit d'un model de fluid sota tràfic intens

Abstract: Es considera un sistema format per una xarxa de diverses estacions de servei amb un servidor a cadascuna i capacitat d'emmagatzematge ilimitada, on es va processant un fluid continu segons va arribant a cada estació. Quan deixa una estació, el fluid pot necessitar passar per una altra estació de servei (*retroalimentació*) o pot abandonar el sistema. Com a procés d'arribades externes es considera un procés no-determinista generat per un gran nombre de fonts que alternen estrictament períodes ON (en els quals envien fluid al sistema a taxa constant) i OFF. A més, es suposa que la distribució de la longitud dels períodes ON i OFF té les cues pesades.
En aquest context, i sota tràfic intens, es demostra que el procés de *"càrrega de treball del sistema"*, fent un canvi d'escala adequat, tendeix en cert sentit cap a un procés que es coneix com a *"moviment Brownià fraccional amb reflexió"*.
També es demostra una Llei Fleble dels Grans Nombres Funcional (FWLLN) per a dos processos associats al sistema: la quantitat de fluid que arriba a les estacions (incloent les arribades externes i les degudes a la retroalimentació), i la quantitat de fluid que surt de les estacions (cap a d'altres, o deixant el sistema).

17/01/2007 Pierre Vuillermot, Université Henri Poinvaré, Nancy I, Nancy, France
Variational solutions to a class of parabolic partial differential equations driven by a fractional noise

Abstract: In this talk I will present some recent results of D.Nualart and myself concerning the existence and the uniqueness of certain variational solutions to a class of nonautonomous semilinear stochastic partial differential equations of parabolic type defined on bounded open subsets of Euclidean

space of arbitrary dimension and driven by an infinite multiplicative fractional noise. I will also briefly indicate the main ingredients of the proofs, which rest upon the convergence of a suitable Faedo-Galerkin scheme. In particular, I will show that this convergence can be obtained through a new compact embedding theorem whose proof relies on the use of a classic result of Hardy, Littlewood and Pólya regarding the boundedness properties in Lebesgue spaces of certain integral transforms with homogeneous kernels.

31/01/2007 Tomás Caraballo, Universidad de Sevilla, Spain
Synchronization of a stochastic reaction-diffusion system on a thin two-layer domain

Abstract: We consider a system of semi-linear parabolic stochastic partial differential equations with additive space-time noise on the union of thin bounded tubular domains $D_{1,\varepsilon} := \Gamma \times (0, \varepsilon)$ and $D_{2,\varepsilon} := \Gamma \times (-\varepsilon, 0)$ joined at the common base $\Gamma \subset \mathbb{R}^d$ where $d \geq 1$. The equations are coupled by an interface condition on Γ which involves a reaction intensity $k(x', \varepsilon)$, where $x = (x', x_{d+1}) \in \mathbb{R}^{d+1}$ with $x' \in \Gamma$ and $|x_{d+1}| < \varepsilon$. Random influences are included through additive space-time Brownian motion, which depend only on the base spatial variable $x' \in \Gamma$ and not on the spatial variable x_{d+1} in the thin direction. Moreover, the noise is the same in the both layers $D_{1,\varepsilon}$ and $D_{2,\varepsilon}$. The aim of the talk is to obtain some limiting properties of the global random attractor as the thinness parameter of the domain $\varepsilon \to 0$, i.e. as the initial domain becomes thinner, when the intensity function possesses the property $\lim_{\varepsilon \to 0} \varepsilon^{-1} k(x', \varepsilon) = +\infty$ In particular, the limiting dynamics is described by a single stochastic parabolic equation with the averaged diffusion coefficient, and nonlinearity term, which essentially indicates synchronization of the dynamics on both sides of the common base Γ. Moreover, in the case of non-degenerate noise we obtain stronger synchronization phenomena in comparison with analogous results in the deterministic case.

14/02/2007 Josep Vives, Universitat Autònoma de Barcelona, Bellaterra, Spain
On the short-time behavior of the implied volatility for jump-diffusion models with stochastic volatility

Abstract: In this paper we use Malliavin calculus techniques to obtain an expression for the short-time behavior of the at-the-money implied volatility skew for a generalization of the Bates model, where the volatility does not need to be neither a diffusion, nor a Markov process. The expression depends on the derivative of the volatility in the Malliavin sense. Finally we apply our result to different volatility models. (This is a joint work with Elisa Alòs, Dpt. d'Economia i Empresa, Universitat Pompeu Fabra. and Jorge A. León

Dpt. de Control Automático, CINVESTAV IPN, México D.F.

José Fajardo, IBMEC Business School, Rio de Janeiro, Brasil
Symmetry and Skewness Premium in Lévy Markets

21/02/2007 Andrey A. Dorogovtsev, Institute of Mathematics, National Academy of Science of Ukraine, Kiev, Ukraine
Anticipating integration for Gaussian processes, boundary value problems ans stochastic flows

Abstract: The talk is devoted to the stochastic anticipating equations with the extended stochastic integral with respect to the Gaussian processes of a special type. Such integrals are the generalizations of the well-known Skorokhod integral for a wide class of the Gaussian processes, which includes, for example, the fractional Brownian motion. In the particular cases the solutions of above mentioned equations are the well-known Wiener functionals after the second quantization. As an application, the stochastic Kolmogorov equation for the conditional distributions of the diffusion process is obtained. Also we will consider the conditional variant of the Feynman–Kac formula. The obtained results are apllied to the investigation of the additive functionals from Arrattia flow.

07/03/2007 Gabor Lugosi, ICREA, Universitat Pompeu Fabra, Barcelona, Spain
Local tail bounds for functions of independent random variables

Abstract: In this joint work with Luc Devroye it is shown that functions defined on $\{0, 1, \cdots, r-1\}^n$ satisfying certain conditions of bounded differences that guarantee subgaussian tail behavior, also satisfy a stronger "local" subgaussian property. For self-bounding and configuration functions, we derive analogous locally subexponential behavior. The key tool is Talagrand's (1994) variance inequality for functions defined on the binary hypercube which we extend to functions of uniformly distributed random variables defined on $\{0, 1, \cdots, r-1\}^n$ for $r = 2$. Several applications are also presented.

14/03/2007 José Manuel Corcuera, Universitat de Barcelona, Barcelona, Spain
Jump power assets: a tool for optimization and hedging in a Lévy Market

Abstract: In our market model the stock price process $S = \{S_t, t \geq 0\}$ is a geometric Lévy process $\frac{\mathrm{d}S_t}{S_{t-}} = b\mathrm{d}t + \mathrm{d}Z_t$, $S_0 > 0$,where b is a constant and $Z = \{Z_t, t \geq 0\}$ is a Lévy process.
Except for the geometric Brownian model and the geometric Poissonian model, for the above described general geometric Lévy market models are

incomplete. In this talk, following [1] and [2], we complete the market by a series of assets related to the power-jump processes of the underlying Lévy process. By the completeness of the enlarged market we obtain the optimal portfolio by the martingale method. We will see also how the artificial assets, mentioned aboved, can be related with call options with different strikes, showing how the Lévy market can be completed by using complex porfolios that include call options. This talk is based basically in [3].

[1] Corcuera, J.M., Nualart, D., Schoutens, W. (2003) Completion of a Lévy Market by Power-Jump-Assets. *Finance and Stochastics* **9**(1), 109-127.

[2] Corcuera, J.M.; Nualart, D.; Schoutens, W. (2005) Moment derivatives and Lévy-type market completion. In: Option pricing and Advanced Lévy models. Wilmott Collection. Chichester: Wiley.

[3] Corcuera, J.M.; Guerra, J.; Nualart, D.; Schoutens, W. (2006) Optimal investment in a Lévy market *Applied Mathematics and Optimization,***53**: 279-309.

21/03/2007 Noèlia Viles, Universitat Autònoma de Barcelona, Bellaterra, Spain

Continuity with respect to the Hurst parameter of the laws of the multiple fractional integrals

Abstract: We prove the weak convergence in $\mathcal{C}([0,T])$ of the laws of the Itô and Stratonovich multiple integrals of some classes of deterministic functions with respect to the fractional Brownian motion, B^H, with Hurst parameter $H > 1/2$, to the law of the corresponding multiple integral with respect to B^{H_0}, when H tends to $H_0 \in [1/2, 1)$. This is a joint work with Maria Jolis.

11/04/2007 Iván Torrecilla-Tarantino, Universitat de Barcelona, Barcelona, Spain

Large Deviation Principle in Hölder Norm for Multiple Fractional Integrals

Abstract: For a fractional Brownian motion B^H with Hurst parameter $H \in]\frac{1}{4}, \frac{1}{2}[\cup]\frac{1}{2}, 1[$, multiple indefinite integrals on a simplex are constructed and the regularity of their sample paths are studied. Then, it is proved that the family of probability laws of the processes obtained by replacing B^H by $\varepsilon^{\frac{1}{2}} B^H$ satisfies a large deviation principle in Hölder norm. The definition of the multiple integrals relies upon a representation of the fractional Brownian motion in terms of a stochastic integral with respect to a standard Brownian motion. For the large deviation principle, the abstract general setting given by Ledoux in [Lecture Notes in Math., vol. 1426 (1990) 1-14] is used. This is a joint work with Marta Sanz-Solé.

18/04/2007 Ole E. Barndorff-Nielsen, Aarhus University, Aarhus, Denmark
Upsilon Transformations

Abstract: Upsilon transformations may be viewed as endomorphisms either on the space $\mathfrak{M}_L\left(\mathbb{R}^d\right)$ of m-dimensional Levy measures or on the space $ID\left(\mathbb{R}^d\right)$ of infinitely divisible distributions on $\mathbb{R}^d$ or as a class of stochastic integrals of the form $\phi \bullet L$ where L is a Lévy process and ϕ is deterministic. In wide generality these mappings are smoothing injections and they commute. There are interesting connections to classical families of ID laws, to free probability, and to log infinite divisibility.

25/04/2007 Dan O. Crisan, Imperial College, London, UK
On a new class of weak approximations for solutions of SDE and their applications to the filtering problem

Abstract: In the eighties, Kusuoka and Stroock showed that some of the smoothness properties of the semigroup associated to the solution of a stochastic differential equation are preserved even when the SDE is degenerate. They showed that the semigroup remains smooth in any direction belonging to the Lie algebra associated to the SDE as long as a certain condition, called the UFG condition, is satisfied. This qualitative result gave rise to a number of numerical algorithms for solving SDE developed in the last five years by Lyons, Kusuoka, Ninomiya and Victoir. I show that these algorithms have certain common characteristics and analyze their properties. I also present an application of these methods to the approximation of the solution of Zakai equation.
The talk is based on joint work with S. Ghazali.

06/06/2007 David Márquez-Carreras, Universitat de Barcelona, Barcelona, Spain
L'equació de la calor fraccional: un altre punt de vista

Abstract: En aquest seminari tractarem l'anomenada equació de la calor fraccional. En l'operador d'aquesta equació hi participa una potència positiva del Laplacià negatiu $(-\Delta)^{\gamma/2}$. Si l'equació està governada per un soroll correlacionat en espai podem donar sentit a aquests tipus d'equacions per qualsevol potència positiva.
A part d'ensenyar alguns resultats assolits en un treball conjunt amb la Carme Folrit també m'agradaria donar una àmplia literatura sobre el tema, presentar sense entrar en molts detalls la justificació del cas determinista i oferir algunes diferències amb l'equació d'ona.

06/06/2007 Monique Pontier, Université Paul Sabatier, Toulouse, France
Volatility jump detection

Abstract: We consider a model with piecewise constant volatility, the intra jump times are independant random variables. We propose estimators of the volatility jump number, the volatility jump times and the intra jump volatilities. We obtain convergence in probability of these estimators when the observation number is going to infinity.

14/06/2007 David Nualart, University of Kansas, Lawrence, USA
Stochastic heat equation and intersection local times

Abstract: We discuss the d-dimensional heat equation driven by a multiplicative Gaussian noise, which is white in space and it has the covariance of the fractional Brownian motion with Hurst parameter H in time. First we consider the equation in the Ito-Skorohod sense, and we show that the mild solution has an explicit development in the Wiener chaos. We provide sufficient conditions on the dimension d and the Hurst parameter H for the solution to exist. We prove that in the case $H > 1/2$ the square norms of the projections on each Wiener chaos are related to the moments of some weighted intersection local times of the d-dimensional Brownian motion. On the other hand, using and approximation argument and Feynman-Kac's formula we obtain an expression for the moments of the solution in terms of the exponential moments of the weighted intersection local times. We also consider the case of Stratonovich-type equations.

15/06/2007 Sebastián del Baño, RSB Global Banking and Markets, London, UK
Trading volatility and a problem in stochastic processes

Abstract: In this talk we present a problem on the quadratic variation of certain stochastic processes originating in the trading of volatility derivatives. We will begin by a refresher on the concept of the implied and local volatility surface for a financial asset and its connection with the terminal PDF of spot. Then we present a modest taxonomy of known volatility products, variance swaps, tunnel variance swaps, volatility swaps, covariance swaps, correlation swaps, forward volatility agreements, etc. These are particularly popular amongst the Hedge fund community in their search for alpha. We will also present some term sheets and market quotes and describe some practical issues.
Amongst these products the variance swap is completely understood thanks to the work of Emanuel Derman and only depends on the smile to expiration of the contract. We will briefly describe its pricing. The problem we wish to present occurs in the pricing of volatility swaps, a product which is dependent on the joint PDF's of spot rather than only on the marginals. Based on market empirics and heuristics we believe the value of volatility swaps to

be maximized for certain stochastic dynamics. This is then translated into a problem of maximizing the expected quadratic variation of martingales subject to certain constraints.